Some Business Problems of American Forestry
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PREFACE.

FORESTRY on a large scale will not be possible in the United States, on private holdings, unless it proves to be a remunerative investment of capital.

Unfortunately, owing to the slowness of tree growth, there is no chance of large profits in forestry. Not one of the forest owners abroad has engaged in forestry with a view of getting rich through it. Forestry is not a maker of wealth; it is only a preserver of wealth.

As a preserver of wealth, forestry is unrivalled. No business yields interest on the capital engaged in it as steadily as forestry. As sure as the sun shines, the wind blows and the rain falls, the volume of a tree is compelled to increase, the increment representing the interest on tree-capital. Sunshine, air and precipitations are the factors of tree growth.

Abroad, all aristocratic families owning forest estates have succeeded in the maintainance of their standing for centuries, whilst the rich merchants and bankers of olden times have not left a trace of their names and their wealth, in spite of the fact, that many of them, the merchants of Leipzig, Hamburg, Midland, Nuernberg, were richer than their sovereigns themselves.

In America, it is the well-to-do class, and pre-eminently the well-to-do lumberman, who should be interested in forestry, wherever it offers him a safe and remunerative chance of investment.

The "Problems of Forestry" were compiled with a view of showing the American wood owners the financial character of professional forestry. The object in forestry, as in any other business, is the production of high and safe interest on capital. Some little knowledge of elementary mathematics and of banking generally is required for the solution of any financial problem. The banker, the insurance company, the stock broker, constantly meet with tasks similar to those outlined in the "Problems of Forestry."

The splendid interest tables issued by the Mutual Life Insurance Company were used for the solution of the problems in order to avoid the times-taking application of logarithms.

C. A. SCHENCK,
Forester to the Biltmore Estate.

Biltmore, N. C., March, 1900.
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A LONGLEAF PINE PROBLEM (FLORIDA).

PREMISES: Mr. S., of E., Florida, owns a pine forest of all ages, so that seedlings, saplings, poles and trees are equally mixed, and estimates that the annual growth is 250 feet board measure per acre. The tract is 100,000 acres and he thus cuts 25,000,000 feet board measure annually with the view of not decreasing the growing stock. The expense for taxes and the cost of protection from fire, etc., is 5c per acre per annum; the value of the stumpage is $1 per thousand feet board measure. Mr. S. thinks that the quality of the forest will be improved gradually, and expects an increase in productiveness, of one per cent. annually. He figures, besides, on rising stumpage prices, the rise keeping step with the increase in population (1⅔ per cent.). He has a chance to invest money at 5 per cent. in an equally safe manner and wants to sell the forest.

QUESTION: Below what price per acre is it not advisable for Mr. S. to sell?

POINTS:
1. Mr. S. must figure at 5 per cent. interest, as the equally safe investment promises him 5 per cent. as well.
2. If the productiveness of the forest increases by 1 per cent. per annum, and the stumpage price at 1⅔ per cent. per annum, the receipts will grow at the rate of 2½ per cent. per annum. In discounting these receipts backwards, we have to figure at 5 per cent. —2½ per cent. = 2¼ per cent.
3. The present value of all annual receipts is $25,000
   \[ \frac{25,000}{0.050 - 0.025} \]
4. The present value of all expenses (taxes and protection) is
   \[ \frac{100,000 \times 0.05}{0.05} \]
   \[ \frac{25,000}{0.050 - 0.025} - \frac{100,000 \times 0.05}{0.05} = X \]
   RESULT: $900,000 for the whole forest, or $9 per acre.
ANOTHER LONGLEAF PINE PROBLEM (FLA.)

PREMISES: Near Pensacola, Florida, a tract of 50,000 acres stocked with longleaf pine, not boxed for turpentine, is for sale, the stumpage averaging 3000 feet board measure per acre. The present owner offers the stumpage alone at $1 per thousand feet, or else is willing to sell the fee simple (soil and trees together) at $3.50 per acre.

Under conservative lumbering, the annual production of timber is 133 feet board measure per acre, to be drawn from a growing stock of 1500 feet board measure per acre. The land itself is practically unfit for farming. No damage is to be feared from forest fires, as long as turpentine is not obtained from the forest. Figure at 6 per cent. interest, and at 1c taxes per acre per annum.

QUESTION: Is it advisable for a Pensacola mill firm, of twenty-five million feet annual capacity, to buy the stumpage alone, or is it more profitable for it to acquire stumpage and land together with a view of practicing forestry?

POINTS:
1. When buying the fee simple, the firm can cut, within three years, 1500 feet per acre—the biggest trees—leaving the balance of 1500 feet on the ground, and obtaining from that balance henceforth on an annual average as much as the annual accretion, namely 133 feet board measure (worth 13.3 cents) per acre.
   2. Thus the firm obtains from the forest
      (a) For three years annually 500 x 50,000 feet board measure worth $25,000.
      (b) From the fourth year on, annually 133 x 50,000 feet board measure worth $6,650.
3. The expense for taxes will, for the whole tract, amount to $500 annually.
4. The firm paying $3.50 x 50,000 equal to $175,000 cash obtains assets worth $25000 (1.06^3—1) + $6650 = $500

   EQUATION: Entrepreneur's Gain
   \[
   \frac{25000 \times 1.06^3 - 1}{0.06 \times 1.06^3} + \frac{6650}{1.06^3 \times 0.06} - \frac{500}{0.06} = 175000
   \]
   \[
   = 25000 \times 2.67 + 6650 \times 14.0 - 500 \times 16.7 = 175000
   \]
   \[
   = 66750 + 93100 = 83500 - 175000
   \]

RESULT: Bent on forestry, the firm seemingly incurs an undertaker’s loss of about $25,000, paying more for the forest than the forest is able to refund.

If, however, the prices of stumpage can be expected to rise by 50 per cent. in the course of the next 20 years (corresponding with a
rise of 2 per cent. per annum), acquisition of the fee simple at $3.50 per acre implies a net gain of about $33,600.

In addition it must be remembered that the tract, under destructive lumbering, cannot be logged over in less than six years, the mill capacity being 25 million feet board measure only.

Thus the firm, when engaging in ordinary lumbering, pays in fact more than $1.00 per 1000 feet board measure, namely

\[
\frac{150,000 \times 0.06 \times 1.06^6}{25,000 (1.06^6 - 1)} = 1.22
\]

If this consideration holds good, acquisition of fee simple and practice of forestry is preferable even under stagnating prices, being by about $11,700 superior to destructive lumbering.
A RED FIR PROBLEM (OREGON).

PREMISES: Mr. W——r, a Michigan lumberman, had a chance to acquire, in 1862, 200,000 acres of splendid white pine forest, at 40c per acre. He had made 7 per cent. on the investment, and a total net gain of $1,200,000 when the last tree was cut in 1888.

In 1900 Mr. W——r moved to Oregon, and had a chance in the Cascade backwoods to exactly repeat the speculation of 1862, buying 200,000 acres of splendid Douglas fir, scaling 30,000 feet board measure per acre, at 40c per acre.

The value of the denuded land, in Michigan and Oregon, is nill. Taxes 1c per acre per annum.

QUESTION: What must be the development of stumpage prices in the section referred to, if Mr. W——r is again to make a net gain of $1,200,000 in addition to deriving 7 per cent. from the investment, in the course of 26 years?

POINTS:
The answer depends on the rate, at which the railroad system in the section will be developed, by the establishment of which the gradual removal of the timber will be made possible.

I. Suppose cutting begins after 16 years, in 1916, and ends in 1926.

1. The average cut will be 600 million feet board measure per annum, worth 600,000X. The receipts for stumpage, in 1926, will have accumulated to $600,000 X (1.07^{10} - 1)

2. The accrued expense for taxes will be, at 1c per acre per annum, $2000 (1.07^{26} - 1)

if we assume, that taxes on the whole land are paid until 1926.

3. The original price paid for the land, $80,000, has grown up, at 7 per cent. compound interest, to year 1926, to $80,000 X 1.07^{26}.

EQUATION:

\[
1,200,000 = \frac{600,000 X (1.07^{10} - 1)}{0.07} - \frac{2000 (1.07^{26} - 1)}{0.07} - 80,000 \times 1.07^{26}
\]

RESULT: The stumpage price of Oregon pine, in that section, must average about 22 cents per 1000 feet board measure, from 16 years from to-day on.

II. Suppose cutting begins after 10 years, in 1910, and ends in 1926.

1. The average cut will then be 6,000,000,000 = 375 million feet
board measure per annum. The receipts from stumpage in 1926 will have accumulated to 
\[375,000 \times (1.07^{16} - 1)\]

2. and 3. as under 1. 

\[\frac{0.07}{0.07}\]

EQUATION:

\[1,200,000 = \frac{375,000 \times (1.07^{16} - 1) - 2000 \times (1.07^{26} - 1) - 80,000 \times 1.07^{26}}{0.07}\]

RESULT: The stumpage price of Oregon pine, in that section, must average about 17 cents per 1000 feet board measure, from 10 years from to-day on.
A YELLOW POPLAR PROBLEM (N. C.).

PREMISES: A careful tally of the yellow poplar trees, standing in the valley drained by Claw Hammer creek, furnishes the following data:

1090 trees of \( \frac{1}{2} \) foot diameter, containing 300 feet b. m. each, equal to 327,000 feet b. m.

1980 trees of 2 foot diameter, containing 450 feet b. m. each, equal to 891,000 feet b. m.

860 trees of \( 2\frac{1}{2} \) foot diameter, containing 700 feet b. m. each, equal to 602,000 feet b. m.

680 trees of 3 foot diameter, containing 1000 feet b. m. each, equal to 680,000 feet b. m.

270 trees of over \( 3\frac{1}{2} \) foot diameter, containing 1700 feet b. m. each, equal to 459,000 feet b. m.

The stumpage of these trees is worth, on an average, $0.75 per 1000 feet b. m. in the case of trees of \( \frac{1}{2} \) foot diameter, 1.25 per 1000 feet b. m. in the case of trees of 2 foot diameter, 2.50 per 1000 feet b. m. in the case of trees of \( 2\frac{1}{2} \) foot diameter, 4.25 per 1000 feet b. m. in the case of trees of over \( 3\frac{1}{2} \) foot diameter, 6.25 per 1000 feet b. m.

The trees, up to \( 2\frac{1}{2} \) foot in diameter, are growing at a rate of \( \frac{1}{2} \) inch in 10 years, and thereafter at a rate of 1 inch in 10 years.

The taxes are not apt to be reduced after the timber is cut.

The owner believes, that the price of poplar stumpage will double in 25 years (annual increase of \( 2\frac{1}{2} \) per cent), and is satisfied to make 5 per cent on the investment.

QUESTION: At what rate shall the owner dispose of the trees, viz. down to what diameter limit is it advisable for him to sell the trees?

POINTS:

1. A tree of \( \frac{1}{2} \) foot diameter, in 40 years, attains 2 foot diameter.
A tree of 2 foot diameter, in 40 years attains \( 2\frac{1}{2} \) foot diameter.
A tree of \( 2\frac{1}{2} \) foot diameter, in 60 years attains 3 foot diameter.
A tree of 3 foot diameter, in 60 years attains \( 3\frac{1}{2} \) foot diameter.

2. The \( \frac{1}{2} \) foot tree, now worth $0.75 \times 0.300 = 22\frac{1}{2} \) cents, will be worth after 40 years $1.25 \times 0.450 \times 1.025^{40} = \ldots \$ 1.52.
The 2 foot tree, now worth $1.25 \times 0.450 = 56 \) cents, will be worth after 40 years $2.50 \times 0.700 \times 1.025^{40} = \ldots \$ 4.72.
The \( 2\frac{1}{2} \) foot tree, now worth $2.50 \times 0.700 = 175 \) cents, will be worth after 60 years $4.25 \times 1.000 \times 1.025^{60} = \ldots \$ 18.70.
The 3 foot tree, now worth $4.25 \times 1.000 = 425 \) cents, will be worth after 60 years $6.25 \times 1.700 \times 1.025^{60} = \ldots \$ 46.80.

3. All trees, growing at a 5 per cent. rate, are to remain; those growing at a rate of less than 5 per cent. will be cut.
EQUATIONS:
For 1½ foot trees: \[ 0.225 \times 1.0^{X^{10}} = 1.52; \ X = 5 \ \text{per cent.} \]
For 2 foot trees: \[ 0.562 \times 1.0^{X^{10}} = 4.72; \ X = 5\frac{1}{2} \ \text{per cent.} \]
For 2½ foot trees: \[ 1.750 \times 1.0^{X^{10}} = 18.70; \ X = 4 \ \text{per cent.} \]
For 3 foot trees: \[ 4.250 \times 1.0^{X^{10}} = 46.80; \ X = 4 \ \text{per cent.} \]
RESULT: The owner should dispose of all trees having a diameter of 2½ foot or more, as they grow only at a rate of 4 per cent. Obviously, unsound trees which are apt to deteriorate, should be cut at once whatever the diameter be.
ANOTHER YELLOW POPLAR PROBLEM (N.C.)

PREMISES: Pisgah forest contains 40,000 acres, stocked with 60 million feet board measure yellow poplar of superior quality, worth now $3.50 per thousand feet board measure. The owner expects that the prices of yellow poplar stumpage will double within the next 15 years (increase of 5 per cent. per annum), and that then small logs and defective logs will have a value as well, so that 70 million feet board measure will be available in the year 1915.

The taxes and the general expenses take six cents per acre per annum.

The value of the soil, after the timber is cut, can be assumed to be $2 per acre.

The owner figures at 6 per cent. interest.

QUESTION: What is the profit from the investment, if any at the end of the next 15 years, aside from the interest of 6 per cent?

POINTS:
1. The present value of the investment is $60,000 x 3.50 for the trees and $40,000 x 2.00 for the soil.
2. The value of the forest in 1915 is $70,000 x 7.00 for the trees and $40,000 x 2.00 for the soil.
3. The running expenses from 1900 to 1915 are, per annum, $0.06 x 40,000. They accumulate up to 1915, to the sum

EQUATION: \[ X = \frac{70,000 \times 7.00 + 40,000 \times 2.00 - 0.06 \times 40,000 (1.06^{15} - 1)}{0.06} - \frac{1.06^{15}(60,000 \times 3.50 + 40,000 \times 2.00) - 0.06 \times 40,000 (1.06^{15} - 1)}{0.06} \]

RESULT: The owner will find himself $182,000 short. He will lack a good deal from making 6 per cent. on his investment. As a matter of fact, he will make about 4 per cent. on the investment and no more, unless the stumpage prices do more than double within the next 15 years.
AN ADIRONDACK PROBLEM.

PREMISES: A tract of land in the Adirondacks, acquired in the year 1876 at $5 per acre, was cut over in 1888, yielding then, per acre, 1800 feet b. m. White pine, worth $3 per thousand feet b. m., and 2600 feet b. m. Spruce, worth $1 per thousand feet.

In the year 1896, there were cut per acre another 6550 feet b. m. of spruce, worth $1.50 per thousand feet b. m.

The taxes on the forest were 5c per acre per annum; the expense of administration and protection 2c per acre per annum. Figure at 6 per cent.

QUESTION: At what cost were those last 6550 feet b. m. produced?

POINTS:
1. The price paid for the land, in 1876, was $5 per acre, which accrued, at compound interest, and up to the year 1896, to $5.00 \times 1.06^{20}

2. The running expenses, during the period 1876 to 1896, were 7 cents per acre per annum, and sum up to the amount of

$$\frac{0.07 \times (1.06^{20} - 1)}{0.06}$$

3. The yield made in 1888 was $3.00 \times 1.8 + $1.00 \times 2.6 = $8.00 Discounted forward to the year of calculation, 1896, this yield, (which is of course to be subtracted from the various outlays) amounts to $8.00 \times 1.06^8.$

EQUATION:

$$X = 5 \times 1.06^{20} + \frac{0.07 \times (1.06^{20} - 1)}{0.06} - 8 \times 1.06^8$$

RESULT: The cost of producing those 6550 feet was $5.80. As the value of the 6550 feet is $9.82, the owner has gained, aside from making 6 per cent. interest on the investment and aside from having the value of the culled forest for an additional asset, about $4.00 per acre.
A SPRUCE PROBLEM.

PREMISES: A spruce forest contains in the year 1899 per acre about 4228 feet b. m., consisting of trees scaling 10 inches or more at four feet from ground.

First case: Suppose that the owner actually cuts down to 10 inches diameter, thus removing now 4228 feet. Then, in the year 1929 he will be able to again cut 2420 feet b. m., cutting again down to 10 inches. Every 34 years thereafter he will have the same yield of 2420 feet b. m. The price of the stumpage is $1.40 per thousand feet. The taxes are, on an average, 3c per acre per annum.

Second case: Suppose the owner cuts only down to 12 inches. Then the forest will yield, to begin with, 3608 feet b. m. In the year 1919 the owner will be able to cut 2115 feet b. m., and thereafter the same amount every 24 years. The value of stumpage in this case is $1.50 per thousand feet; the taxes are, on an average, 4c per acre per annum.

Third case: Suppose the owner cuts only down to 14 inches. Then the first yield will be, in the year 1899, 2846 feet b. m. In the year 1919 the owner will be able to cut 2624 feet b. m., and thereafter every 21 years the same amount. The value of the stumpage is $1.60 per thousand feet; the taxes are, on an average, 5c per acre per annum.

The owner wants to derive 5 per cent. interest and believes in a gradual increase of the prices, the increase to average 2 per cent. per annum. The cost of administration and protection is in all cases 5c per acre.

QUESTION: Had the owner of the forest better cut down to 10 inches, to 12 inches or to 14 inches?

POINTS:
1. The maximum difference between expected yields and expected expenses determines the best course to be followed.
2. In the 10 inch case the yields are
   \[ 4,228 \times 1.40 + \frac{1.03^{4} \times 2,420 \times 1.40}{1.03^{34} - 1} = 8.14 \]
   In the 12 inch case the yields are
   \[ 3,608 \times 1.50 + \frac{1.03^{4} \times 2,115 \times 1.50}{1.03^{24} - 1} = 8.80 \]
   In the 14 inch case the yields are
   \[ 2,846 \times 1.60 + \frac{1.03 \times 2,624 \times 1.60}{1.03^{21} - 1} = 9.56 \]
3. In the 10 inch case the expenses are \( \frac{0.03}{0.05} = 1.60 \)
In the 12 inch case the expenses are \[ \frac{0.04}{0.05} + \frac{0.05}{0.05} = \$1.80 \]

In the 14 inch case the expenses are \[ \frac{0.05}{0.05} + \frac{0.05}{0.05} = \$2.00 \]

\text{EQUATION:} \quad (8.14 - 1.60) \overset{\leq}{\sim} (8.86 - 1.80) \overset{\geq}{\sim} (9.56 - 2.00)

\begin{align*}
6.54 & \quad 7.06 & \quad 7.56 \\
\text{RESULT:} & \quad \text{Cutting down to 14 inches only, the most conservative practice, pays best. It pays by} & \quad \$
\end{align*}

\begin{align*}
\text{by} & \quad \text{by} & \quad \text{better than cutting down to 10 inches, and by} & \quad 50c. \text{ per acre, better than cutting down to 12 inches.}
\end{align*}
ANOTHER SPRUCE PROBLEM.

PREMISES: A lumberman owns 20,000 acres of spruce land, from which he has just cut 6000 feet board measure per acre, 12 inch and over in diameter at the stump, worth $1.50 per thousand. After another 20 years he will be able to obtain 3320 feet per acre, cutting again down to 12 inch diameter, and we may expect, that, after 40 years, the same yield will be obtainable and so on.

The land, when cleared, is said to have some value for pasture purposes. The taxes are 4 cts., the expense for administration, protection, etc., 8 cts. per acre per annum. Figure at 6 per cent.

QUESTION: What is the forest worth at the present moment?

POINTS:
1. After 20, 40, 60 (and so on) years, a yield of 3320 feet b.m., worth $4.98 can be obtained.
2. The necessary expenses are 4 cents plus 8 cents per acre per annum.
3. The value of a forest, like the value of a house or a farm or a business is equal to the present value of all returns, minus all expenses, expected from it.

EQUATION: \[ X = \frac{4.98}{1.06^{20} - 1} \frac{0.12}{0.06} = 2.22 - 2.00 \]

RESULT: The forest, after lumbering, is worth 22 cts per acre.

If the owner can sell it, for farming purposes, at over 22 cts. per acre, he should certainly do it, provided that he can make, by re-investing the proceeds of the sale, 6 per cent. in an equally safe manner.

If the taxes, or the expense necessary for administration, protection, etc., are 2 cts higher per acre per annum than is supposed in the premises, the owner had better give up the land after the first cutting, unless he can sell it, for in that case its forestry value is negative, the necessary expenses devouring all possible profits.

If, on the other hand, there is a good chance for the stumpage prices to rise, say at the average rate of 2 per cent. per annum, the cut over forest has a value of

\[ \frac{4.98}{1.04^{20} - 1} \frac{0.12}{0.06} = \$2.15 \text{ per acre.} \]

The study of future prices of stumpage is of the very greatest importance for the wood-owner.
A WHITE PINE PROBLEM (MINNESOTA.)

PREMISES: A Minnesota lumberman owns 10,000 acres of white pine forest, containing 6000 feet b. m. per acre, worth $3 per thousand. The agricultural value of the land is $5 per acre, when the timber is removed. Under conservative lumbering, an annual production of 300 feet b. m. per acre can be expected. Taxes 8cts per acre per annum. Protection from fire, under forestry, 12cts per acre per annum. Extra logging expenses, under forestry, $4 per acre, at the first cutting. Lumber prices expected to double in 35 years (= annual rise of 2 per cent.) Proper growing stock for forestry 2000 feet b. m. per acre.

QUESTION: What interest on the investment will forestry yield?

POINTS:
1. The investment, to begin with, is 6000 feet b. m. worth $3 = $18 per acre plus value of soil worth $5 per acre.
2. The yield under forestry is 4000 feet worth $3 = $12 per acre to be derived at once, and 300 feet worth 90cts to be derived annually thereafter, being the annual production of the 2000 feet left standing, per acre. The future yields are to be discounted at (X per cent. — 2 per cent.)
3. The annual expenses, under forestry, are 20 cents. The extraordinary expenses are $4 per acre, spent at the first cutting.

EQUATION: \( 18 + 5 = 12 + \frac{90}{0.0X - 0.02} - \frac{.20}{0.0X} - 4 \)

RESULT: About 7 per cent.
A SHORT LEAF PINE PROBLEM (ARK.)

PREMISES: The S. & A. Lumber Co., of B., Arkansas, owns 100,000 acres of forest stocked, per acre, with 6000 feet board measure merchantable short leaf pine, and has a mill of thirty million feet board measure annual capacity. The stumpage is worth $1 per 1000 feet board measure. The land is unfit for agriculture. Under conservative cutting the forest will continuously produce 200 feet board measure per acre per annum, after the “Virgin Surplus” of the forest, consisting per acre of 4500 feet of hypermature and mature trees, has been removed. Under conservative cutting, the logging expenses are 10 cents higher per thousand feet board measure. Prices are expected to rise at i 1-2 per cent per annum. Figure at 6 per cent. Taxes are 1ct per acre per annum. Protection from fire, under conservative lumbering, will cost 3cts per acre per annum.

QUESTION: Which pays better, conservative or exhaustive lumbering?

POINTS:
1. If prices rise at i 1-2 per cent., future yields must be discounted back to the present moment at 6 per cent — i 1-2 per cent equal to 4 i-2 per cent.
2. Under exhaustive lumbering, the forest will yield 30,000,000 feet b. m. for 20 years, and nothing more. The land, being non-agricultural, will be thrown away after 20 years. The value of all yields expected from the forest, minus taxes for 20 years, is therefore:

\[
\frac{30000 \left(1.045^{20} - 1\right)}{0.045 \times 1.045^{20}} - \frac{1000 \left(1.06^{20} - 1\right)}{0.06 \times 1.06^{20}}
\]

3. Under conservative lumbering, we withdraw as well annually 30 million feet from the forest, as long as the mature stock of 4500 x 100,000 equal to 450 million feet lasts, namely for 15 years.
4. From the 16th year on, we cut only the annual production, namely, 200 x 100,000 equal to 20 million feet per annum.
5. As the extra logging expenses, in this case, are 10 cents higher, the timber has a stumpage value of 90 cents only, instead of $1 per 1000 feet board measure.
6. Taxes and protection from fire will cost annually 0.04 x 100,000 equal to $4000.
7. Thus, under conservative management, the present value of all expected yields, minus expenses for taxes and protection is:

\[
\frac{27000 \left(1.045^{15} - 1\right)}{0.045 \times 1.045^{15}} + \frac{18000}{0.045 \times 1.045^{15}} - \frac{4000}{0.06}
\]

18
EQUATION:

\[
\frac{30000 (1.045^{20} - 1)}{0.045 \times 1.045^{20}} - \frac{1000 (1.06^{20})}{0.06 \times 1.06^{20}} = \frac{27000 (1.045^{15} - 1)}{0.045 \times 1.045^{15}} + \frac{18000}{0.045 \times 1.045^{15}} - \frac{4000}{0.06}
\]

\[
30000 \times 13.0 - 1000 \times 11.5 > 27000 \times 10.7 + 400,000 \times 0.5 = 66,700
\]

RESULT: Conservative lumbering pays, by about $43,700, better than exhaustive lumbering.
INFLUENCE OF FOREST FIRES ON RATE OF INTEREST.

PREMISES: Absolutely safe investments (f. i. U. S. bonds) yield about 3 per cent. interest on the principal.

Forestry in America is a less safe investment. Of course, trees live and grow as sure as the sun shines, the wind blows and the rain falls, for sunshine, air and rainwater are the components of wood fibre. Still the ravages of forest fires endanger both capital and returns from capital.

In the Southern Alleghanies, 2 per cent. of the woodlands, on an average, are annually damaged by fire. Nature will require 20 years for the restoration of the burned forest to its former value and productiveness.

QUESTION: What is the minimum rate of annual interest which forest-growth, under these conditions, must yield?

POINTS:
1. 98 acres out of 100 acres are left intact, 2 acres out of 100 acres suffer a reverse resulting in a setback of 20 years.
2. Hence the value of those 2 acres is reduced to $\frac{1}{1.0X^{20}}$ of what it was to begin with.
3. The growth of every 98 units of value left intact must make up for the loss through burning in such a way, as to bring the value of the total investment, at the end of the year, up to 103.
4. The 98 develop into the value 98 x 1.0X. This value, plus what remains of the 2 units damaged by fire, must be 103 if the investor shall make 3 per cent. on the original principal.

EQUATION: $98 \times 1.0X \times \frac{1}{1.0X^{20}} = 103$

RESULT: 4 1-4 per cent. Unless those sections of the forest, which are left intact, grow at a rate of 4 1-4 per cent., the owner does not make 3 per cent. of absolute interest on the whole investment. It will pay the owner to annually spend up to 1 1-4 per cent. of the investment for forest protection, if by such expensive precaution fires can be entirely avoided.

Where fires result, on the annual average, in the entire destruction of 5 per cent. of the forest, the annual production on areas left intact must amount to at least 8.4 per cent. if the entire investment is to bring 3 per cent. interest. Such a production is impossible, unless the price of standing timber doubles within the next 15 years.
STUMPAGE PRICES OF THE FUTURE.

PREMISES: Virgin forest is unproductive, because, annually, just as much timber is lost, through death and decomposition of trees, as there is produced under the influence of sun, rain and wind.

Cut over forest is, as a general rule, almost unproductive, owing to fires following the removal of the virgin growth.

Hence it is safe to say, that the annual production of timber in the 700 million acres of American woodlands is not over one-fifth of what it might be, (namely, 200 feet board measure per acre) and is not likely to exceed 40 feet board measure per annum per acre, or 28 billion feet board measure on the whole.

The total growing stock of timber in the United States is estimated to be 900 billion, and the annual consumption is estimated to be 39 billion feet board measure, (exclusive of firewood).

It is expected that, after the exhaustion of the American virgin surplus of timber, prices of stumpage will be at a level with those prevailing in No-Surplus countries, where, f. i., prime white oak stumpage is worth $75 instead of $3 here, and prime pine stumpage is worth $15 instead of $1.50 here, per thousand feet board measure.

QUESTION: At what annual rate can we expect the stumpage prices of oak and pine to rise, whilst our surplus is being gradually exhausted?

POINTS:
1. The annual consumption of timber exceeds the annual production by at least 11 billion feet.
2. Hence our surplus stock of 900 billion will be consumed in \( \frac{900}{11} \) equal to 82 years.
3. After 82 years, stumpage of oak will be worth about 25 times, and stumpage of pine will be worth about 10 times of what it is worth now.

EQUATION: Oak: \( 3.00 \times 1.0 \times X^{82} = 75.00 \)
           Pine: \( 1.50 \times 1.0 \times X^{82} = 15.00 \)

RESULT: The price of oak stumpage can be expected to rise at a rate of about 4 per cent. per annum; the price of pine stumpage at a rate of about 3 per cent. per annum.
FOREST TAXATION IN THE UNITED STATES.

PREMISES: In America taxes depend on the value of property. In the less densely settled sections, where most of our forests are situated, the taxes amount to 1 per cent. of the property value.

Given a forest, which when 90 years old contains $20 worth of timber per acre. The soil alone, when cleared, is worth only $1 per acre. Rate of interest 4 per cent.

QUESTION: What taxes ought to be justly imposed upon the owner,—if taking 1 per cent. of the forest value is just taxation—at the 60th and 30th year of the development of the forest, and further, in the year, in which the seedlings were just starting?

POINTS:

1. The taxes being 1 per cent. of the value of the forest, that value—a prospective value—must be ascertained for the years 60, 30 and 0.

2. The forest 90 years old is worth $21.00—$0.21, the 21 cents being the amount of taxes due in the year 90,

\[ \frac{21.00 - 0.21}{100} \times 21 \]

The forest 89 years old is thus worth

\[ \frac{99}{100} \times 21 \]

equal to

\[ \left( \frac{99}{100} \right)^2 \times \frac{21}{1.04} \]

The forest 88 years old is thus worth

\[ \frac{99}{100} \times \frac{21}{1.04^2} \]

The forest 87 years old is thus worth

\[ \frac{99}{100} \times \frac{21}{1.04^3} \]

3. The forest 60 years old is thus worth

\[ \frac{99}{100} \times \frac{21}{1.04^{30}} \]

4. The forest 30 years old is thus worth

\[ \frac{99}{100} \times \frac{21}{1.04^{60}} \]

5. The forest 0 years old is thus worth

\[ \frac{99}{100} \times \frac{21}{1.04^{90}} \]

EQUATIONS:

\[
\text{Taxes in forest 60 years old} = \frac{1}{100} \left( \frac{99}{100} \right)^{31} \times \frac{21}{1.04^{30}} \\
\text{Taxes in forest 30 years old} = \frac{1}{100} \left( \frac{99}{100} \right)^{61} \times \frac{21}{1.04^{60}} \\
\text{Taxes in forest 0 years old} = \frac{1}{100} \left( \frac{99}{100} \right)^{91} \times \frac{21}{1.04^{90}}
\]

RESULT: At 60 years 4 7-10 cent.

At 30 years 1 1-10 cent.

At 0 years 2-10 cent.
INFLUENCE OF TAXES ON BUSINESS FORESTRY.

PREMISES: The premises are just as in foregoing problem.

QUESTION: To what figure do the taxes (paid from the first to the 90th year of the forest) accumulate up to the time at which the timber is mature, namely, up to the 90th year?

POINTS:
1. Taxes in the year 0 of the forest are
   \[ \frac{1}{100} \left( \frac{99}{100} \right)^{91} \frac{21}{1.04^{90}} \] which, discounted to
   the year 90, amount to \[ \frac{21}{100} \left( \frac{99}{100} \right)^{91} \]

2. Similarly, the taxes paid in the year 1 of the forest accrue to
   \[ \frac{21}{100} \left( \frac{99}{100} \right)^{90} \] and the taxes paid in the year 2 of the forest accrue to \[ \frac{21}{100} \left( \frac{99}{100} \right)^{89} \] and so on.

3. Thus the discounted values of the taxes form a geometrical progression of 91 numbers, with \[ \frac{99}{100} \] for the constant factor.

EQUATION: Sum of taxes equal to

\[
\frac{21}{100} \left\{ \frac{1 - \left( \frac{99}{100} \right)^{91}}{1 - \left( \frac{99}{100} \right)} \right\} = 21 \left\{ \frac{1 - \left( \frac{99}{100} \right)^{91}}{1 - \left( \frac{99}{100} \right)} \right\}
\]

RESULT: The sum of taxes paid, accrued at compound interest is $12.60, and thus takes away 60 per cent. of the final yield obtainable.

It is especially noteworthy, that the rate of interest used does not influence the accrued sum of taxes in the least. The sum simply depends upon the number of years required to develop merchantable timber out of seedling trees.

It is safe to say that, under the conditions now prevailing in America, the taxes consume over one-half of the value of the yield derivable from forestry, if they are "justly" imposed. No wonder, then, that people are averse of engaging in forestry.

If the rotation is 150 years, the taxes curtail the final yield by 77 per cent!
A NATIONAL PARK PROBLEM (MINNESOTA).

PREMISES: It is proposed to establish a national park in Northern Minnesota, on land unfit for farming, stocked with about 6000 feet board measure pine timber per acre, worth $2.50 per thousand feet board measure.

Four thousand feet board measure, per acre on an average, consists of mature or hypermature timber, whilst the balance is thrifty, and can be expected to yield by its own growth and by giving rise to a second growth, 150 feet board measure per acre per annum, under conservative management.

In this case the first logging expenses will by 50 cents per acre exceed the logging expenses under destructive lumbering.

Taxes, protection from fire and administration will take 10 cents per acre per annum. Five and one-half per cent. interest. — Stumpage prices expected to double in 35 years (corresponding with an annual increase of 2 per cent.)

QUESTION: Will conservative lumbering, in this case, pay better than destructive lumbering?

POINTS:
1. Under destructive lumbering, the value of the forests is 6000 X 2.50 equal to $15 per acre.
2. Under conservative lumbering, 4000 feet are cut at once, and 2000 feet are left standing as a permanent investment, yielding annually 0.150 X 2.50 equal to $0.375 per acre, from which figure, however, the annual expenses of 10 cents must be deducted.

EQUATION: 15 > 10.00 - 0.50 + 0.275 = 0.035.

RESULT: Under conservative lumbering, an entrepreneur’s gain of about $2.50 per acre is obtained. Hence conservative lumbering pays better.

Unless the first logging expenses under forestry are by $3.00 per acre higher than is usually the case, destructive lumbering is financially inferior to conservative lumbering.

After destructive lumbering, the land is left worthless. Under conservative lumbering $10 are withdrawn from the forest, and there remains a permanent asset of about $8 per acre on the ground, yielding 5½ per cent. interest per annum.
STATE LOANS FOR FORESTRY PURPOSES.

PREMISES: The state of P. decides to engage in state forestry, and to that end takes up a loan of $1,450,000, at 4 per cent., which is to be used as follows:

$1,000,000 for purchase of 1,000,000 acres at the headwaters of the rivers,

$ 50,000 for lawyer's fees, surveys, demarkation, etc.,

$ 150,000 for roads, etc.,

$ 250,000 for defraying the annual salaries of superintendents and rangers. After the road system is developed, 12 years from today, an annual revenue of 10 cents per acre per annum will be derived, under conservative management, and it is expected that this revenue will gradually increase at the rate of 3 per cent. per annum.

QUESTION: A: Within how many years will the forest itself be able to redeem the loan?

B: What is the forest worth after the loan is redeemed?

POINTS:

1. The annual interest on the loan is $58,000.

2. The revenue from the 12th year on is 0.10 x 1,000,000 = $100,000, of which $58,000 are used, from that time on, to pay the interest on the loan, whilst $42,000 or more are available annually for redemption of loan.

3. As the revenue rises at 3 per cent. per annum, the receipts are to be discounted at 4 per cent. — 3 per cent. = 1 per cent.

EQUATION:

A: \[ \frac{42000 \times (1.01^x - 1)}{0.01 \times 1.01^x} = 1,450,000 \]

B: \[ \frac{100,000 \times 1.03^4}{0.01} = X \]

RESULT: A: The loan can be entirely redeemed within 43 years after the 12th year, or within 55 years from today.

B: The forest, freed from all incumbrances after 55 years, and producing annually $356,000 with prospect of an annual increase of revenue equalling 3 per cent., is worth about $35,600,000.

If the people are ready to spend $58,000 for 12 years, they will gradually build up a very valuable forest.

Far-leading calculations in forestry might seem fallacious and absurd, if forest-history had not proved the very contrary.
WEEDING AND ROAD-BUILDING.

PREMISES: Given a spruce forest in the Adirondacks, containing 50,000 acres stocked with 4,000 feet b. m. per acre. The forest is to be cut over at once in such a way as to reduce the average stumpage to 1,200 feet b. m., which are expected to produce thereafter 140 feet b. m. per acre per annum. Stumpage is worth $1.50 per 1,000 feet b. m. Cuttings recur every 10 years.

QUESTION: A: Is it advisable for the owner, to remove, by way of an additional "weeding," the misshapen trees at an expense of 50 cents per acre, if by so doing the annual production can be raised from 140 feet to 175 feet b. m. per acre?

B: Is it advisable for the owner to build the skidding roads more solidly, at an expense of $10,000 so as to make them available for future operations, and so as to be less dependent on the snow-covering, if by so doing the logging expenses per 1000 feet b. m. can be reduced by 5cts?

POINTS: A: The additional expense of 50 cents results in producing, every ten years, an additional 350 feet b. m. worth 52½ cents.

B: The additional expense of $10,000, reduces the cost of logging, and hence increases the value of stumpage, by 5cts per 1000 feet. The first cut, therefore, of 50,000 x 2800 = 140,000,000 feet b. m., gains $7,000; the subsequent cuttings, of 50,000 x 1750 = 87,500,000 feet b. m., gain $4,375.

EQUATION: A: \[0.50 = \frac{0.525}{1.0 \times 10^1 - 1}\]

B: \[10,000 = \frac{7,000 + \frac{4375}{1.0 \times 10^1 - 1}}{1.0 \times 10^1 - 1}\]

RESULT: The "weeding" as well as the solid construction of roads are advisable, the former paying 7 per cent. and the latter 9½ per cent. interest on the additional outlay required. The advantage derived is greater still, if stumpage prices are on the increase.